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$\mathbb{Z}_N$ orientifolds with flux

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Abstract: We compute the flux induced tadpole and superpotential in various type IIB $\mathbb{Z}_N$ compact orientifolds in order to study moduli stabilization. We find supersymmetric vacua with $g_s < 1$ and describe brane configurations with cancelled tadpoles. In some cases moduli are only partially fixed unless anti D3-branes are included.

Keywords: D-branes, Superstring Vacua.

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Contents

1. Preamble

A most attractive feature of compactifications with fluxes is the existence of a moduli dependent potential with minima at finite values. Flux induced moduli stabilization in type IIB orientifolds on T^6, $\mathrm{K3}\times\mathrm{T}^2$, $\mathrm{T}^6/\mathbb{Z}_2\times\mathbb{Z}_2$, and Calabi-Yau threefolds, has been largely discussed in recent times [1]–[13]. In this note we consider type IIB $\mathcal{N}=1$ orientifolds with internal space $\mathrm{T}^6/\mathbb{Z}_N$, studied only briefly up to now [6], which in fact provide a simpler setup in which moduli stabilization can be described in detail. Moreover, full models with tadpole cancellation can be constructed. We will focus on the behavior of complex structure moduli and the dilaton-axion $\tau = C_0 + ie^{-\phi}$. We have no new insights about fixing Kähler moduli so they will be dropped from the analysis.

In Type IIB compactifications it is natural to turn on fluxes of the NS-NS and R-R 3-forms. These fluxes must satisfy the quantization conditions

$$\frac{1}{(2\pi)^2\alpha'}\int_\gamma F_3 \in \mathbb{Z}\,; \qquad \frac{1}{(2\pi)^2\alpha'}\int_\gamma H_3 \in \mathbb{Z}\,, \tag{1.1}$$

for any 3-cycle γ in the internal space $\mathcal{M}$. It is useful to introduce the combination $G_3 = F_3 - \tau H_3$. The equations of motion require that G_3 be imaginary self-dual (ISD), i.e. ${}^*G_3 = iG_3$ [14].

The fluxes induce a tadpole for the R-R 4-form with coefficient

$$N_{\text{flux}} = \frac{1}{(2\pi)^4\alpha'^2}\int_{\mathcal{M}} H_3 \wedge F_3\,. \tag{1.2}$$

In $\mathbb{Z}_N$ toroidal orientifolds with orientifold action including a reflection I_6 of all six internal coordinates there are 64 O3-planes, and typically D3-branes, that contribute to the tadpole. We consider only O3-planes with neither NS-NS nor R-R backgrounds, thus with charge $-1/2$ (in units such that a D3-brane has charge 1). This requires that in the flux quantization (1.1) all integers be even [2, 3, 6]. The C_4 tadpole cancellation condition is then

$$N_{\mathrm{D}_3} + \widetilde{N}_{\mathrm{flux}} = 32\,, \tag{1.3}$$

where N_{D_3} is the net number of D3-branes. Here $\widetilde{N}_{\mathrm{flux}}$ is computed on the torus, this is appropriate because the number of dynamical 3-branes is not $32/2N$. In general there are twisted tadpoles whose cancellation requires a non-zero number of D3-branes pinned down at the origin, i.e. the point fixed by the full orientifold group. For N even there are further O7-planes and D7-branes and the C_8 tadpole must be cancelled.

The fluxes also generate a superpotential [15]

$$W = \int_{\mathcal{M}} G_3 \wedge \Omega\,, \tag{1.4}$$

where Ω is the holomorphic (3,0) form. The superpotential depends on the dilaton and the complex structure moduli denoted U_α. The ISD condition on G_3 is equivalent to demanding [14]

$$D_\tau W \equiv \partial_\tau W + (\partial_\tau K)W = 0\,; \qquad D_\alpha W \equiv \partial_\alpha W + (\partial_\alpha K)W = 0\,, \tag{1.5}$$

where

$$K = -\log[-i(\tau - \overline{\tau})] - \log\left(-i\int_{\mathcal{M}} \Omega \wedge \overline{\Omega}\right) \tag{1.6}$$

is the tree-level Kähler potential for τ and U_α. We will not include Kähler moduli in the analysis. However, we implicitly assume that their Kähler potential is of no-scale form so that (1.5) corresponds to the minimum of the supergravity potential. Generically W does not vanish at the minimum, thus supersymmetry is broken by the F-term of the Kähler moduli. This soft supersymmetry breaking is measured by $\exp\mathcal{G}$, where $\mathcal{G} = K + \log|W|^2$.

As usual we define complex coordinates $z^a = x^a + ix^{a+3}$, $a = 1, 2, 3$, on which the $\mathbb{Z}_N$ action is $\theta : z^a \to e^{2i\pi v_a} z^a$. To preserve supersymmetry we specifically choose $\sum_a v_a = 0$. The torus lattice is denoted Λ, and its basis e_i, $i = 1, \ldots, 6$. Since Λ must be $\mathbb{Z}_N$ symmetric, its deformation parameters are restricted, being actually in one-to-one correspondence with the untwisted Kähler and complex structure moduli [16, 17]. In fact, in these orbifolds there is at most one untwisted U modulus, allowed when N is even and, say, $v_3 = -1/2$.

We use conventions in which

$$\Omega = \lambda\, dz^1 \wedge dz^2 \wedge dz^3\,. \tag{1.7}$$

Since the x^i have flat metric δ_{ij}, Ω is imaginary anti-self-dual, i.e. ${}^*\Omega = -i\Omega$. The normalization factor λ generically has units of (length)$^{-3}$ and will be chosen so that K and W do not depend on the Kähler moduli. This is just a matter of convenience. The physically relevant quantity $\mathcal{G}$ is independent of λ, and the Kähler moduli. When N is even, and $v_3 = -1/2$, there is also an untwisted invariant (2,1) form, namely

$$\xi = \lambda\, dz^1 \wedge dz^2 \wedge d\bar{z}^3\,. \tag{1.8}$$

Notice that ξ is ISD.

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In general, F_3 and H_3 are linear combinations of all $b_3 = 2 + 2h_{12}$ 3-forms but in this note we only turn on fluxes along untwisted 3-forms such as Ω, ξ, and their complex conjugates. Such fluxes are also primitive. The Kähler form, $J = \sum_a J_a dz^a \wedge d\bar{z}^a$, satisfies $J \wedge \Omega = 0$ and $J \wedge \xi = 0$.

2. Examples

In this section we determine the flux induced quantities in various $\mathrm{T}^6/\mathbb{Z}_N$ orientifolds. We will discuss to what extent the fluxes fix the values of the moduli and comment on the effect in the open string spectrum.

We restrict to $\mathbb{Z}_N$ orbifolds in which there is no contribution to h_{21} from twisted sectors. In this situation, the symplectic basis of 3-cycles does not include fractional ones that might require flux integers to be multiple of some minimal quantum [5]. We only need demand even integers in order to avoid exotic O3-planes as mentioned before.

2.1 $\mathrm{T}^6/\mathbb{Z}_3$

The torus lattice has nine free parameters, all corresponding to untwisted Kähler moduli. To simplify we choose Λ to be the product of three SU(3) root lattices (up to size) orthogonal to each other. This means $e_i \cdot e_{i+1} = -R_i^2/2$, $i = 1, 3, 5$, and all other mixed $e_i \cdot e_j = 0$. A suitable normalization for Ω is $\lambda^{-1} = R_1 R_3 R_5$. Since $h_{21} = 0$, there are only two homology 3-cycles and two harmonic 3-forms, namely Ω and $\overline{\Omega}$. Hence we can write

$$F_3 = a_F \overline{\Omega} + \text{c.c.}\,; \qquad H_3 = a_H \overline{\Omega} + \text{c.c.}\,. \tag{2.1}$$

To determine the coefficients we must impose the quantization conditions (1.1) and to this end we need a basis of 3-cycles.

As in [18] we define the toroidal 3-cycles

$$\pi_{ijk} = e_i \otimes e_j \otimes e_k\,, \tag{2.2}$$

where the e_i are the 1-cycles corresponding to the lattice basis. Invariant combinations are systematically found by taking $\mathbb{Z}_N$ orbits of the π_{ijk} [18]. Given the $\mathbb{Z}_3$ action, $\theta e_i = e_{i+1}$, $\theta e_{i+1} = -e_i - e_{i+1}$, $i = 1, 3, 5$, one readily checks that there are just two invariant independent 3-cycles. A convenient basis is provided by the orbits of π_{135} and π_{136}. Explicitly,

$$\begin{aligned} \gamma_1 &= -\pi_{136} - \pi_{145} - \pi_{146} - \pi_{235} - \pi_{236} - \pi_{245} \\ \gamma_2 &= \pi_{136} + \pi_{145} + \pi_{235} + \pi_{135} - \pi_{246}\,. \end{aligned} \tag{2.3}$$

Besides, $\gamma_1 \cap \gamma_2 = 1$ so that the basis is symplectic. The periods of Ω, i.e. the integrals $Y_A = \int_{\gamma_A} \Omega$, are easily calculated to be $Y_1 = 1$ and $Y_2 = e^{2i\pi/3}$. Using (1.1) we then obtain

$$a_H = \frac{1}{2\sqrt{3}}[\sqrt{3}\ell_1 + i(2\ell_2 + \ell_1)]\,; \qquad \ell_1, \ell_2 \in \mathbb{Z}\,. \tag{2.4}$$

For a_F, just replace ℓ by k. To avoid cluttering we omit factors of $(2\pi)^2\alpha'$ that can be reinserted in the end. An equivalent way to proceed is to expand the fluxes in the dual cohomology basis Σ_A such that $\int_{\gamma_B}\Sigma_A = \delta_A^B$. Then, $F_3 = k^A\Sigma_A$ and $H_3 = \ell^A\Sigma_A$. From (2.4) one can easily deduce the Σ_A in terms of Ω and $\overline{\Omega}$ and compute the intersection $\eta_{12} = -1$, where $\eta_{AB} = \int_{\mathcal{M}}\Sigma_A\wedge\Sigma_B$.

For the flux induced tadpole we find

$$\widetilde{N}_{\rm flux} = 3(k_1\ell_2 - k_2\ell_1)\,. \tag{2.5}$$

Clearly, $\widetilde{N}_{\rm flux} = 3\ell^A\eta_{AB}k^B$. The factor of 3 takes into acount that $\widetilde{N}_{\rm flux}$ is computed on the torus. The superpotential is $W = i\sqrt{3}(a_H\tau - a_F)$. Imposing the ISD condition as $D_\tau W = 0$, or as cancellation of the coefficient of Ω in G_3, gives

$$\tau = \frac{2k_2 + k_1 + i\sqrt{3}k_1}{2\ell_2 + \ell_1 + i\sqrt{3}\ell_1}\,. \tag{2.6}$$

Notice that requiring ${\rm Im}\tau > 0$ implies $\widetilde{N}_{\rm flux} > 0$. Recall that the flux integers are even, thus $\widetilde{N}_{\rm flux}$ is quantized in multiples of 12.

Another constraint on the fluxes is the validity of the perturbative expansion. Using (2.5) and (2.6) we can write the string coupling $g_s = e^\phi$ as

$$g_s = \frac{2\sqrt{3}(\ell_2^2 + \ell_1^2 + \ell_2\ell_1)}{\widetilde{N}_{\rm flux}}\,. \tag{2.7}$$

This immediately shows that there are no solutions with $\widetilde{N}_{\rm flux} = 12$ and $g_s < 1$. Furthermore, there is basically just one solution with $\widetilde{N}_{\rm flux} = 24$ and $g_s < 1$. For instance, using a vector notation, the fluxes $\ell = (0, 2)$, $k = (4, -2)$, fix the dilaton at $\tau_0 = i\sqrt{3}$, and at the minimum $|W_0| = 4\sqrt{3}$. All other choices are the same up to ${\rm SL}(2,\mathbb{Z})$ transformations. With larger $\widetilde{N}_{\rm flux}$ the number of solutions with $g_s < 1$ obviously increases. For instance, for $\widetilde{N}_{\rm flux} = 48$ there are vacua with $g_s = \sqrt{3}/6$ and $g_s = \sqrt{3}/2$. However, according to (1.3), anti D3-branes must be added to absorb the extra charge, thus breaking supersymmetry in an explicit way.

In the $\mathbb{Z}_3$ orientifold with $\Omega' = \Omega(-1)^{F_L}I_6$, twisted tadpole cancellation requires that at least 8 D3-branes be placed at the origin O. This can be seen directly from the condition ${\rm Tr}\,\gamma_{\theta,3,O} = -4$, where $\gamma_{\theta,3,O}$ is the embedding of the orbifold action on the Chan-Paton labels. In turn this condition can be obtained applying T-duality to the regular Ω orientifold with D9-branes in which ${\rm Tr}\,\gamma_{\theta,9} = -4$ [19, 20]. The smallest such γ_θ, with $\gamma_\theta^3 = \mathbb{1}$, is of dimension 8.

An alternative view is the following. In the regular Ω orientifold the D9 gauge group is ${\rm SO}(8)\times{\rm U}(12)$, which can be broken by Wilson lines [20]. In particular, there is a family of discrete Wilson lines that gives ${\rm SO}(8-2n)\times{\rm U}(12-2n)\times{\rm U}(n)^3$, $n = 0, \cdots, 4$. In the T-dual picture the first two factors arise from $(32-6n)$ D3-branes staying at the origin while the last factor is due to $3n$ branes located at some other fixed point P of θ, for which ${\rm Tr}\,\gamma_{\theta,3,P} = 0$ (the remaining $3n$ branes are at the image of P under I_6). Since all branes are placed at $\mathbb{Z}_3$ fixed points the rank of the group is not reduced. Observe that we can remove at most 24 D3-branes from O. These could be arranged as just described or they could be sent fully to the bulk as 4 dynamical 3-branes that could be traded by flux.

The upshot is that with $\widetilde{N}_{\rm flux} = 24$ all tadpoles can be cancelled by placing 8 D3-branes at the origin. They have gauge group U(4) and three chiral multiplets in the **6**. This chiral content is clearly anomaly free.

With lower $\widetilde{N}_{\rm flux}$ more interesting brane configurations can be designed. A 3-family SU(5) model with $\widetilde{N}_{\rm flux} = 12$ and 20 D3-branes is described in [6].

2.2 $\mathrm{T}^6/\mathbb{Z}_7$

The torus lattice has three Kähler moduli encoded in the R_i entering in the lattice vectors [16]. As Ω normalization we take $\lambda^{-1} = R_1R_3R_5$. In this case $h_{21} = 0$ so the fluxes are of the form (2.1). Using $\theta e_i = e_{i+1}$, $i = 1,\ldots,5$, $\theta e_6 = -e_1 - e_2 - e_3 - e_4 - e_5 - e_6$, shows that there are just two invariant independent 3-cycles. As basis we take γ_1, γ_2 equal to the orbits of π_{123} and π_{124} respectively. These fulfill $\gamma_1 \cap \gamma_2 = 1$. The periods of Ω are $Y_1 = -i\sqrt{7}$ and $Y_2 = (7 + i\sqrt{7})/2$. It then follows

$$\begin{aligned} a_H &= \frac{1}{14}(2\ell_2 + \ell_1 - i\sqrt{7}\ell_1)\,, \\ \widetilde{N}_{\rm flux} &= 7(k_1\ell_2 - k_2\ell_1)\,. \end{aligned} \tag{2.8}$$

The superpotential is $W = i7\sqrt{7}(a_H\tau - a_F)$. The ISD condition leads to

$$\tau = \frac{2k_2 + k_1 + i\sqrt{7}k_1}{2\ell_2 + \ell_1 + i\sqrt{7}\ell_1}\,. \tag{2.9}$$

As before, $\mathrm{Im}\tau > 0$ implies $\widetilde{N}_{\rm flux} > 0$. Without anti D3-branes the maximum $\widetilde{N}_{\rm flux}$ is 28. There is just one vacuum with $g_s < 1$, achieved choosing, say $\ell = (0,2)$, $k = (2,0)$. Then, $\tau_0 = (1 + i\sqrt{7})/2$, $|W_0| = 14$.

According to (1.3), to cancel the untwisted tadpole, 4 D3-branes are needed. Twisted tadpole cancellation requiring $\mathrm{Tr}\,\gamma_{\theta,3,O} = 4$ is then satisfied placing the branes at the origin. They give rise to group SO(4) without matter. In the regular orientifold, the D9 gauge group $\mathrm{SO}(8) \times \mathrm{U}(4)^3$ can be broken by a discrete Wilson line to $\mathrm{SO}(4) \times \mathrm{U}(2)^7$. In the T-dual picture the SO(4) comes from the 4 branes at the origin and $\mathrm{U}(2)^7$ from 14 branes located at another θ fixed point P, for which $\mathrm{Tr}\,\gamma_{\theta,3,P} = 0$ (the remaining 14 branes being at the orientifold image). The 28 D3-branes liberated from the origin can move completely to the bulk and turn into flux.

The SO(4) group of the D3-branes at the origin is pure $\mathcal{N} = 1$ super Yang-Mills. This raises the possibility of adding a non-perturbative superpotential $W_{np} \sim \exp(8\pi^2 i\tau/C(G))$ generated by gaugino condensation. However, for SO(4) with Casimir $C(G) = 2$, the flux superpotential linear in τ dominates and the solution of $D_\tau(W + W_{np}) = 0$ stays at τ_0. For gaugino condensation to produce a bigger effect one needs groups with a larger Casimir.

2.3 $T^6/\mathbb{Z}_6'$

The $\mathbb{Z}_6'$ action has $v_1 = \frac{1}{6}$, $v_2 = \frac{1}{3}$, $v_3 = -\frac{1}{2}$. It can be realized in four different lattices [17]. Here we work with Λ of SU(6) $\times$ SU(2). All possible lattices allow five real deformation parameters, two of them corresponding to one untwisted complex structure modulus. The number of twisted moduli depends however on the lattice. In our case there are no twisted 3-forms, i.e. $h_{12} = 1$ [17].

The lattice vectors satisfy $\theta e_i = e_{i+1}$, $i = 1, \cdots, 4$, $\theta e_5 = -e_1 - e_2 - e_3 - e_4 - e_5$, $\theta e_6 = -e_6$. Following [16] we write them as

$$\begin{aligned} e_1 &= (R_1, R_3, R_5, 0, 0, 0)\,; & e_4 &= (-R_1, R_3, -R_5, 0, 0, 0) \\ e_2 &= \frac{1}{2}(R_1, -R_3, -2R_5, \sqrt{3}R_1, \sqrt{3}R_3, 0)\,; & e_5 &= \frac{1}{2}(-R_1, -R_3, 2R_5, -\sqrt{3}R_1, \sqrt{3}R_3, 0) \\ e_3 &= \frac{1}{2}(-R_1, -R_3, 2R_5, \sqrt{3}R_1, -\sqrt{3}R_3, 0)\,; & e_6 &= R_6(0, 0, \cos\beta, 0, 0, \sin\beta) \end{aligned} \tag{2.10}$$

The untwisted complex modulus is simply

$$U = \frac{R_6}{\sqrt{3}R_5} e^{i\beta}\,. \tag{2.11}$$

Again, $\lambda^{-1} = R_1 R_3 R_5$.

The most general 3-form background is of type

$$F_3 = a_F \overline{\Omega} + b_F \xi + \text{c.c.}\,; \qquad H_3 = a_H \overline{\Omega} + b_H \xi + \text{c.c.}\,. \tag{2.12}$$

It is easy to show that there are four toroidal invariant 3-cycles. As basis we take γ_A, $A = 1, \cdots, 4$, to be the orbits of π_{123}, π_{124}, π_{126} and π_{136} respectively. The intersection matrix is unimodular. The periods of Ω are $Y_1 = -i\sqrt{3}$, $Y_2 = 3$, $Y_3 = -\sqrt{3}U$ and $Y_4 = -3iU$. To obtain the periods of ξ just replace U by $\overline{U}$. Imposing flux quantization leads to

$$\begin{aligned} a_H &= \frac{1}{12 \mathrm{Im}\, U} \left[\ell_4 - i\sqrt{3}\ell_3 - U(\sqrt{3}\ell_1 + i\ell_2) \right] \\ b_H &= \frac{1}{12 \mathrm{Im}\, U} \left[-\ell_4 - i\sqrt{3}\ell_3 + U(\sqrt{3}\ell_1 - i\ell_2) \right]. \end{aligned} \tag{2.13}$$

The superpotential is $W = 12i\sqrt{3}\mathrm{Im}\, U(a_H \tau - a_F)$. The tadpole coefficient is

$$\widetilde{N}_{\text{flux}} = 6(k_1\ell_4 - k_2\ell_3 + k_3\ell_2 - k_4\ell_1)\,. \tag{2.14}$$

Indeed, the dual 3-forms have intersections $\eta_{14} = -1$, $\eta_{23} = 1$ and $\eta_{AB} = 0$ otherwise. Note that $\widetilde{N}_{\text{flux}}$ is quantized in multiples of 24.

The ISD condition reduces to

$$\begin{aligned} \sqrt{3}k_3 + k_2\overline{U} - \sqrt{3}\ell_3\tau - \ell_2\tau\overline{U} &= 0 \\ k_4 - \sqrt{3}k_1\overline{U} - \ell_4\tau + \sqrt{3}\ell_1\tau\overline{U} &= 0\,. \end{aligned} \tag{2.15}$$

We assume that $\mathrm{Im}\,\tau \neq 0$ and $\mathrm{Im}\, U \neq 0$, i.e. we exclude infinite string coupling and degenerate torus. Then, equations (2.15) can generically be solved to obtain the ratio $\mathrm{Im}\,\tau/\,\mathrm{Im}\, U$, as well as the values of $\mathrm{Re}\,\tau$ and $\mathrm{Re}\, U$. There is a remaining equation quadratic

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in $\mathrm{Im}\,U$ that, depending on the fluxes, might not have real solutions. In fact, for $\widetilde{N}_{\text{flux}} = 24$ we find no solutions with τ and U completely fixed. The best alternative is to obtain a relation between them. Clearly, this can be easily achieved with fluxes such that one of the equations in (2.15) is trivial. For example, with $k = (2,0,0,0)$, $\ell = (0,0,0,2)$, $\tau = -\sqrt{3}\,\overline{U}$. Or with $k = (0,0,2,0)$, $\ell = (0,2,0,0)$, $\tau\overline{U} = \sqrt{3}$.

With $\widetilde{N}_{\text{flux}} = 24$, eq. (1.3) is satisfied by taking $N_{\mathrm{D}_3} = 8$. In fact, one needs 8 D3-branes at the origin to cancel twisted tadpoles. This cancellation condition, as well as the open string spectrum, can be deduced adapting the results in the Ω orientifold with D9 and D5-branes [20]. The D3-branes at the origin have gauge group U(4) with two hypermultiplets in the **6**. There are 32 D7-branes that can be located at the origin thus giving group U(4) $\times$ U(4) $\times$ U(8) with charged hypermultiplets as in the 55 sector [20]. The 37 matter consists of one hypermultiplet in $(\overline{\mathbf{4}};\mathbf{4},\mathbf{1},\mathbf{1}) + (\mathbf{4};\mathbf{1},\overline{\mathbf{4}},\mathbf{1})$. The full matter content is anomaly free.

With larger $\widetilde{N}_{\text{flux}}$ one does find vacua with τ and U fixed. For example, for $\widetilde{N}_{\text{flux}} = 48$ there is the family $k = (2,0,2,2n)$, $\ell = (0,2,0,2)$, $|n| \leq 3$, with $\tau = (n + i\sqrt{12-n^2})/2$, $U = \tau/\sqrt{3}$. Notice that $g_s < 1$ for $|n| \leq 2$. For $|n| = 3$, $g_s > 1$, and τ is fixed under S-duality. For $|n| \geq 4$ these fluxes could only give a solution with $\mathrm{Im}\,\tau = \mathrm{Im}\,U = 0$.

3. Final comments

The purpose of this note was to work out further examples of $D = 4$, $\mathcal{N} = 1$, toroidal orientifolds with fluxes. As we have seen, a rather explicit analysis of moduli stabilization can be carried out. Moreover, D3/D7-brane configurations with cancelled tadpoles, and the corresponding chiral spectrum, can be found. These orientifolds can serve as starting points to build semi-realistic models and study other flux effects such as soft supersymmetry breaking [7] and [21]–[25].

In toroidal orientifolds the fluxes are limited to small values unless anti D3-branes are included to absorb the extra charge. These anti-branes break supersymmetry explicitly, in particular the spectrum is no longer supersymmetric [26]. On the other hand, $\overline{\text{D3}}$-branes could have useful applications as in the proposal to produce de Sitter vacua [27]. Note however that in our case the number of $\overline{\text{D3}}$-branes needed to increase the fluxes appreciably is large, so they are not a small perturbation above a supersymmetric vacuum. Moreover, there are flux-brane anhilation processes that could not be sufficiently suppressed [28].

Another possibility to increase the allowed values of N_{flux} would be to add magnetized D9-branes that can balance the excess charge without breaking supersymmetry as observed recently [29]. Indeed, magnetized D9-branes have been used to construct semi-realistic models in the $\mathrm{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold with fluxes [5]–[7] and [29, 30, 25].

The fluxed orientifolds in this note furnish neat examples in which a counting of vacua could be performed thoroughly. We have concentrated on small fluxes and $g_s < 1$, and found that in this situation there are only very few vacua. In the $\mathbb{Z}_3$ and $\mathbb{Z}_7$ there are no complex structure moduli and the values of τ are indeed determined as in the toy model of

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a rigid Calabi-Yau discussed in [8, 10]. The $\mathbb{Z}_6'$ is the analog of a Calabi-Yau orientifold with one modulus. For large fluxes the number of solutions of (2.15) is expected to agree with general predictions [8]. In the regime of small fluxes one can just analyze (2.15) directly.

Finally, for the $\mathbb{Z}_3$ and $\mathbb{Z}_7$ type IIB orientifolds a perturbative heterotic dual without fluxes is known [19, 31]. It would be interesting to track down the effect of fluxes in the heterotic side.

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References

[1] K. Dasgupta, G. Rajesh and S. Sethi, *M-theory, orientifolds and G-flux*, *J. High Energy Phys.* **08** (1999) 023 [hep-th/9908088].

[2] S. Kachru, M.B. Schulz and S. Trivedi, *Moduli stabilization from fluxes in a simple IIB orientifold*, *J. High Energy Phys.* **10** (2003) 007 [hep-th/0201028].

[3] A.R. Frey and J. Polchinski, $N = 3$ *warped compactifications*, *Phys. Rev.* **D 65** (2002) 126009 [hep-th/0201029].

[4] P.K. Tripathy and S.P. Trivedi, *Compactification with flux on K3 and tori*, *J. High Energy Phys.* **03** (2003) 028 [hep-th/0301139].

[5] R. Blumenhagen, D. Lüst and T.R. Taylor, *Moduli stabilization in chiral type-IIB orientifold models with fluxes*, *Nucl. Phys.* **B 663** (2003) 319 [hep-th/0303016].

[6] J.F.G. Cascales and A.M. Uranga, *Chiral 4D* $N = 1$ *string vacua with D-branes and NS-NS and RR fluxes*, *J. High Energy Phys.* **05** (2003) 011 [hep-th/0303024].

[7] D. Lüst, S. Reffert and S. Stieberger, *Flux-induced soft supersymmetry breaking in chiral type-IIB orientifolds with D3/D7-branes*, hep-th/0406092.

[8] S. Ashok and M.R. Douglas, *Counting flux vacua*, *J. High Energy Phys.* **01** (2004) 060 [hep-th/0307049].

[9] A. Giryavets, S. Kachru, P.K. Tripathy and S.P. Trivedi, *Flux compactifications on Calabi-Yau threefolds*, *J. High Energy Phys.* **04** (2004) 003 [hep-th/0312104].

[10] F. Denef and M.R. Douglas, *Distributions of flux vacua*, *J. High Energy Phys.* **05** (2004) 072 [hep-th/0404116].

[11] A. Giryavets, S. Kachru and P.K. Tripathy, *On the taxonomy of flux vacua*, *J. High Energy Phys.* **08** (2004) 002 [hep-th/0404243].

[12] A. Misra and A. Nanda, *Flux vacua statistics for two-parameter Calabi-Yau's*, hep-th/0407252.

[13] J.P. Conlon and F. Quevedo, *On the explicit construction and statistics of Calabi-Yau flux vacua*, *J. High Energy Phys.* **10** (2004) 039 [hep-th/0409215].

[14] S.B. Giddings, S. Kachru and J. Polchinski, *Hierarchies from fluxes in string compactifications*, *Phys. Rev.* **D 66** (2002) 106006 [hep-th/0105097].

JHEP11(2004)077

[15] S. Gukov, C. Vafa and E. Witten, *CFT's from Calabi-Yau four-folds*, *Nucl. Phys.* **B 584** (2000) 69 [hep-th/9906070].

[16] J.A. Casas, F. Gómez and C. Muñoz, *Complete structure of z(n) yukawa couplings*, *Int. J. Mod. Phys.* **A 8** (1993) 455 [hep-th/9110060].

[17] J. Erler and A. Klemm, *Comment on the generation number in orbifold compactifications*, *Commun. Math. Phys.* **153** (1993) 579 [hep-th/9207111].

[18] R. Blumenhagen, V. Braun, B. Körs and D. Lüst, *Orientifolds of k3 and Calabi-Yau manifolds with intersecting D-branes*, *J. High Energy Phys.* **07** (2002) 026 [hep-th/0206038].

[19] C. Angelantonj, M. Bianchi, G. Pradisi, A. Sagnotti and Y.S. Stanev, *Chiral asymmetry in four-dimensional open- string vacua*, *Phys. Lett.* **B 385** (1996) 96 [hep-th/9606169].

[20] G. Aldazabal, A. Font, L.E. Ibáñez and G. Violero, *$D = 4$, $N = 1$, type-IIB orientifolds*, *Nucl. Phys.* **B 536** (1998) 29 [hep-th/9804026].

[21] P.G. Cámara, L.E. Ibáñez and A.M. Uranga, *Flux-induced SUSY-breaking soft terms*, *Nucl. Phys.* **B 689** (2004) 195 [hep-th/0311241].

[22] M. Grana, T.W. Grimm, H. Jockers and J. Louis, *Soft supersymmetry breaking in Calabi-Yau orientifolds with D-branes and fluxes*, *Nucl. Phys.* **B 690** (2004) 21 [hep-th/0312232].

[23] P.G. Cámara, L.E. Ibáñez and A.M. Uranga, *Flux-induced SUSY-breaking soft terms on D7-D3 brane systems*, hep-th/0408036.

[24] H. Jockers and J. Louis, *The effective action of d7-branes in $N = 1$ Calabi-Yau orientifolds*, hep-th/0409098.

[25] D. Lüst, S. Reffert and S. Stieberger, *MSSM with soft SUSY breaking terms from D7-branes with fluxes*, hep-th/0410074.

[26] G. Aldazabal and A.M. Uranga, *Tachyon-free non-supersymmetric type-IIB orientifolds via brane-antibrane systems*, *J. High Energy Phys.* **10** (1999) 024 [hep-th/9908072].

[27] S. Kachru, R. Kallosh, A. Linde and S.P. Trivedi, *De Sitter vacua in string theory*, *Phys. Rev.* **D 68** (2003) 046005 [hep-th/0301240].

[28] S. Kachru, J. Pearson and H. Verlinde, *Brane/flux annihilation and the string dual of a non-supersymmetric field theory*, *J. High Energy Phys.* **06** (2002) 021 [hep-th/0112197].

[29] F. Marchesano and G. Shiu, *MSSM vacua from flux compactifications*, hep-th/0408059; *Building MSSM flux vacua*, *J. High Energy Phys.* **11** (2004) 041 [hep-th/0409132].

[30] M. Cvetič and T. Liu, *Supersymmetric standard models, flux compactification and moduli stabilization*, hep-th/0409032.

[31] Z. Kakushadze, *Aspects of $N = 1$ type-I-heterotic duality in four dimensions*, *Nucl. Phys.* **B 512** (1998) 221 [hep-th/9704059];
Z. Kakushadze and G. Shiu, *A chiral $N = 1$ type-I vacuum in four dimensions and its heterotic dual*, *Phys. Rev.* **D 56** (1997) 3686 [hep-th/9705163].